ENLIGHT - TRANSMATH meeting: Building networks in Harmonic Analysis and PDEss

Edificio Ensanche, Bilbao (Spain)
11 - 12 November, 2022

Book of Abstracts

Multifractal analysis of Riemann's non-differentiable function

Sergei Iakunin
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Riemann's non-differentiable function (RNDF) is an example of a continuous function which has no derivatives almost everywhere, however it still emerges in some physical phenomena such as dynamics of vortices. The study of RNDF multifractal properties represents a great interest since they may be related to turbulent flows. The most common tool in multifractal analysis of signals is wavelet transform but its application to RNDF is quite delicate due to the complexity of the function. In this talk I will present different methods of multifractal analysis based on wavelet transform and their application to RNDF and solutions of the vortex filament equations.

Harmonic analysis for parabolic boundary value problems and some contributions to mathematical analysis for AI

Kaj Nyström
(Uppsala University, Sweden)

I will give a brief introduction to my research on partial differential equation and harmonic analysis. I will then discuss the parabolic Kato problem and its application to boundary value problems for second order parabolic operators with measurable, non-symmetric and time-dependent coefficients. Finally, I will discuss some contribution to mathematical analysis for AI with a focus on neural networks and PDEs, stochastic gradient descent and low-dimensional manifolds.

From ESPRIT to ESPIRA: Estimation of Signal Parameters by Iterative Rational Approximation

Gerlind Plonka
(Institute for Numerical and Applied Mathematics, University of Göttingen, Germany plonka@math.uni-goettingen.de)

We consider exponential sums of the form

$$f(t) = \sum_{j=1}^{M} \gamma_j \, \mathrm{e}^{\phi_j t} = \sum_{j=1}^{M} \gamma_j \, z_j^t,$$

where $M \in \mathbb{N}$, $\gamma_j \in \mathbb{C} \setminus \{0\}$, and $z_j = \mathrm{e}^{\phi_j} \in \mathbb{C} \setminus \{0\}$ with $\phi_j \in \mathbb{C}$ are pairwise distinct. The recovery of such exponential sums from a finite set of possibly corrupted signal samples plays an important role in many signal processing applications, see e.g. in phase retrieval, signal approximation, sparse deconvolution in nondestructive testing, model reduction in system theory, direction of arrival estimation, exponential data fitting, or reconstruction of signals with finite rate of innovation.

Often, the exponential sums occur as Fourier transforms or higher order moments of discrete measures (or streams of Diracs) of the form $\sum_{j=1}^{M} \gamma_j \, \delta(\cdot - T_j)$ with $T_j \in \mathbb{R}$, which leads to the special case that $\phi_j = \mathrm{i}T_j$ is purely complex, i.e., $|z_j| = 1$.

We introduce a new method for **E**stimation of **S**ignal **P**arameters based on **I**terative **R**ational **A**pproximation (ESPIRA) for sparse exponential sums. Our algorithm uses the AAA algorithm for rational approximation of the discrete Fourier transform of the given equidistant signal values. We show that ESPIRA can be interpreted as a matrix pencil method applied to Loewner matrices. These Loewner matrices are closely connected with the Hankel matrices which are usually employed for recovery of sparse exponential sums. Due to the construction of the Loewner matrices via an adaptive selection of index sets, the matrix pencil method is stabilized. ESPIRA achieves similar recovery results for exact data as ESPRIT and the matrix pencil method (MPM) but with less computational effort. Moreover, ESPIRA strongly outperforms ESPRIT and MPM for noisy data and for signal approximation by short exponential sums.

This talk is based on joint results with N. Derevianko and M. Petz (University of Göttingen).

Riemann's non-differentiable curve

Felipe Ponce-Vanegas
(Basque Center for Applied Mathematics (BCAM), Bilbao, Spain)

What has to do fluid dynamics with quantum mechanics? Vortex filaments are curves around which the vorticity of a fluid is highly concentrated, and in the 70s Hasimoto discovered a bridge that brings solutions between vortex filaments and the non-linear Schördinger equation in 1d. This connection has motivated new questions about the "turbulent" nature of the Schrödinger equation, and everything seems to converge towards the old Riemann's non-differentiable function.

Optimal transport for data analysis and connections to partial differential equations

Bernhard Schmitzer
(University of Göttingen, Germany)

Optimal transport metrics are a geometrically intuitive and robust way of comparing measures with broad applications in analysis, geometry, and statistics. We will give a brief overview on the basic concepts, describe some recent applications in data analysis, and finally sketch current research questions related to partial differential equations.

From the control of the heat equation to spaces of holomorphic functions

Marius Tucsnak
(University of Bordeaux, France)

We present a series of recent developments which lead, in particular, to the full characterization of the reachable space for the boundary controlled heat equation in one space dimension. This progress has been possible due to the establishment of new interactions with the theory of Hilbert spaces of holomorphic functions, which will be described in the talk. We also discuss perturbation results and formulate several open questions.